Chapter 2

Levelling

Aims

After studying this chapter you should be able to:

- Discuss the various types of datum and bench marks that can be used in levelling including Ordnance Datum Newlyn (ODN)
- Describe how automatic, digital and tilting levels work
- Describe the field procedures that are used for determining heights when levelling
- Perform all the necessary calculations and checks for determining heights by levelling including an assessment of the quality of the results obtained
- Appreciate that levelling is subject to many sources of error and that it is possible to manage these
- Outline some methods used in levelling to obtain heights at difficult locations

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2.1 Heights, datums and bench marks

After studying this section you should understand the differences between horizontal and vertical lines or surfaces and why these are important in levelling. You should know what a levelling datum is and be aware that a national levelling datum has been set up by the Ordnance Survey and is known as Ordnance Datum Newlyn (ODN). You should be aware that bench marks are points of known height specified on a chosen datum and that these can either be Temporary or Transferred Bench Marks (TBMs) or Ordnance Survey Bench Marks (OSBMs). In addition, you should appreciate the difficulties of using an OSBM for any levelling and why these are being replaced by GNSS-based heights.

Levelling and how heights are defined

In surveying, three basic quantities are measured – heights, angles and distances – levelling is the name given to one of the methods available for determining heights.

When levelling, it is possible to measure heights within a few millimetres and this order of precision is more than adequate for height measurement on the majority of civil engineering projects. As well as levelling, it is worth noting that heights can also be measured by using total stations, handheld laser distance meters and GNSS receivers – these are described in subsequent chapters of the book. In comparison to these, levelling offers a versatile yet simple, accurate and inexpensive field procedure for measuring heights and this is the reason for its continued use on construction sites in competition with other methods.

The equipment required to carry out levelling is an *optical*, *digital* or *laser level*, which is normally mounted on a *tripod* and used in conjunction with a measuring staff or *levelling staff*. This chapter deals with optical and digital levelling – the techniques used with laser levels to determine and process heights on site are given in Chapter 11 (which covers setting out).

All methods of height measurement determine the heights of points above (or below) an agreed datum. On site or in the office, surveyors, builders and engineers all use, on a daily basis, horizontal and vertical datums as references for all types of measurement including levelling. To illustrate what is meant by horizontal and vertical, Figure 2.1 shows a plumb-bob (a weight on a length of string or cord) suspended over a point P. The direction of gravity along the plumb-line defines the vertical at P and a horizontal line is a line taken at right angles to this. Any horizontal line can be chosen as a datum and the height of a point is obtained by measuring along a vertical above or below the chosen horizontal line. On most survey and construction sites, a permanent feature of some sort is usually chosen as a datum for levelling and this is given an arbitrary height to suit site conditions. The horizontal line or surface passing through this feature, with its assigned height, then becomes the levelling datum. Although it may seem logical to assign a height of 0 m to such a datum, a value often used is 100 m and this is chosen to avoid any negative heights occurring as these can lead to mistakes if the minus sign is accidentally omitted. The heights of points relative to a datum are known as *reduced levels* (RLs).

Any permanent reference point which has an arbitrary height assigned to it or has had its height accurately determined by levelling is known as a *bench mark*. For most surveys and construction work, it is usual to establish the heights of several bench

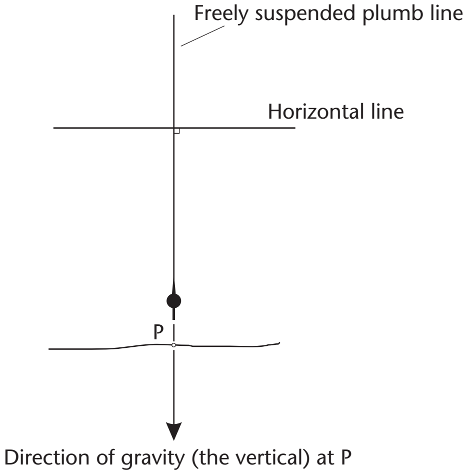

Figure 2.1 ● Horizontal line.

marks throughout a site and if these have heights based on an arbitrary datum, they are known as *Temporary Bench Marks* (*TBM*s).

The positions of TBMs have to be carefully chosen to suit site conditions and various suggestions for the construction of these are given in Chapter 11 and in BS 5964-2:1996 *Building Setting Out and Measurement. Measuring stations and targets*.

The definition of a levelling datum given above is a horizontal or level line or surface that is always at right angles to the direction of gravity. As might be expected, the direction of gravity is generally towards the centre of the Earth and over large areas, because the Earth is curved, a level surface will become curved as shown in Figure 2.2. On this diagram, the height of A above B is measured along a vertical between the level surfaces through A and B.

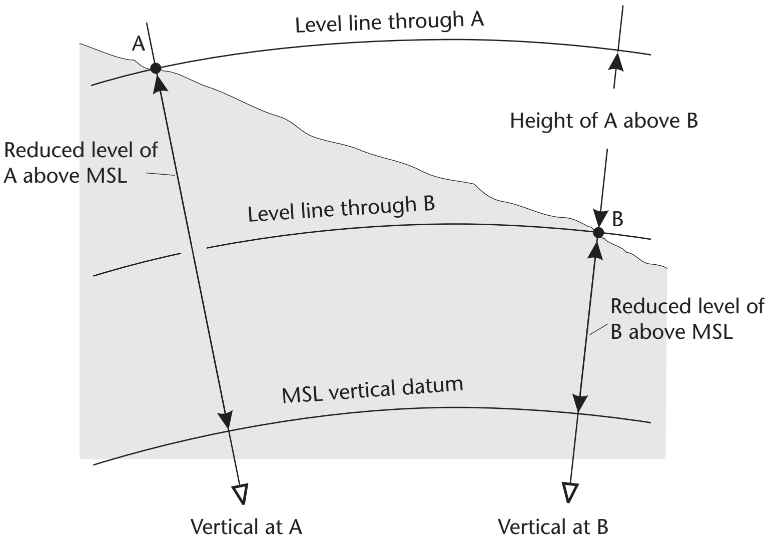

Figure 2.2 ● Level surfaces showing the height difference between two points A and B.

If heights are to be based on the same datum for the whole of a large area such as the UK, a curved level surface of zero height has to be defined. For mainland Great Britain, this has been established by the Ordnance Survey and is known as the *Ordnance Datum Newlyn* (*ODN*) vertical coordinate system. This corresponds to the mean sea level measured at Newlyn, Cornwall and heights which refer to this particular level surface as zero height are known as ODN heights, but are often called heights above mean sea level. Mean sea level (MSL) is represented by a surface known as the *Geoid* which is the level surface to which all height measurements are referenced, whether these are national or local. For more information about Ordnance Datum Newlyn, the Geoid and what mean sea level represents, refer to Chapter 8.

All heights and contours marked on Ordnance Survey maps and plans covering mainland Great Britain are ODN heights and across the country, the Ordnance Survey have established, by levelling, about seven hundred thousand bench marks, known as *Ordnance Survey Bench Marks* (OSBMs) all of which have a quoted ODN height. The most common of these are cut into stone or brick at the base of buildings as shown in Figure 2.3 and the positions and heights of these are shown on some

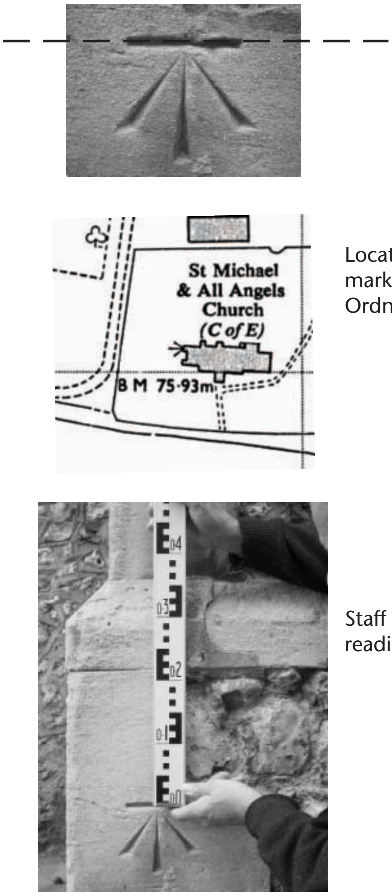

Figure 2.3 ● Ordnance Survey Bench Mark (part reproduced by permission of Ordnance Survey on behalf of the Controller of Her Majesty's Stationery Office, © Crown Copyright 100024463).

Ordnance Survey plans at scales of 1:1250 and 1:2500. Bench mark information is available free of charge from the Ordnance Survey at http://benchmarks.ordnancesurvey.co.uk/.

Some caution must be exercised when using OSBMs as they have not been maintained by the Ordnance Survey since 1989. In fact, many of these have not had their heights revised since the 1970s and they may have been affected by local subsidence or some other physical disturbance since the date they were last levelled. For these reasons, if the heights given on maps or in bench mark lists are to be used for any survey work, it is essential to include two or more OSBMs in levelling schemes so that their height values can be checked for errors by levelling between them. The Ordnance Survey have not been maintaining OSBMs for a number of years because ODN heights of reference points are now determined using GNSS methods. These enable the height of any point at almost any location to be determined so there is now no need to visit an Ordnance Survey bench mark in order to obtain heights relative to mean sea level. The use of GNSS in height measurement is described in Chapters 7 and 8 and in an RICS geomatics guidance leaflet *Virtually level* (this can be downloaded at http://www.rics.org/mappp/).

If the height of a well-defined stable point on a construction site is established by levelling it from a nearby OSBM or GNSS bench mark then this point is known as a *Transferred Bench Mark*, since ODN has been transferred to it. Transferred Bench Marks are also referred to as TBMs and can be used to level other points. The use of Transferred Bench Marks is discussed in Section 11.3.

Despite the existence of OSBMs and GNSS means of realising ODN heights, it is not always necessary to use a mean sea level datum and many construction projects use an arbitrary datum for defining heights.

Reflective summary

With reference to datums and bench marks, remember:

- The Ordnance Survey have established a national vertical datum at mean sea level in mainland Great Britain called Ordnance Datum Newlyn (ODN) – if heights are to be based on this an OSBM has to be used.
- Be careful when using OSBMs as they have not been maintained by the Ordnance Survey for many years – if you need to have ODN heights on site you should use GNSS to do this.
- For most construction work and civil engineering projects, levelling is based on an arbitrary datum and involves using Temporary Bench Marks (TBMs).
- Transferred Bench Marks (also known as TBMs) are well-defined stable points on construction sites which have been levelled relative to ODN, usually from a nearby OSBM or GNSS bench mark.

2.2 Levelling equipment

After studying this section you should be able to describe the differences between tilting, automatic and digital levels. In practical terms, you should know how to set up and use a level, how to handle and read a levelling staff and how to carry out a two-peg test on a level. As well as this, you should understand the operation of a survey telescope, what parallax is and how it can be removed.

This section includes the following topics:

- The levelling staff
- Automatic levels
- Tilting levels
- Digital levels
- Adjustment of the level

The levelling staff

Levelling involves the measurement of vertical distances with reference to a horizontal plane or surface. To do this, a levelling staff is needed to measure vertical distances and an instrument known as a *level* is required to define the horizontal plane. To make it easier to operate, the level is usually mounted on a tripod, as shown in Figure 2.4, which also shows a levelling staff. There are several types of level available and these are discussed in the following pages.

A *levelling staff* is the equivalent of a long ruler and it enables distances to be measured vertically from the horizontal plane established by a level to points where heights are required. Many types of staff are in current use and these can have lengths of up to 5 m. The staff is usually telescopic but can be socketed in as many as five sections for ease of carrying and use and it is made of aluminium or non-conductive fibreglass. The staff markings can take various forms but the E-type staff face recom-

Figure 2.4 ● Levelling equipment (courtesy Leica Geosystems).

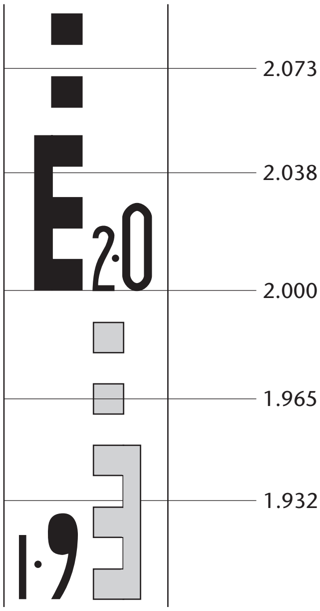

Figure 2.5 ● Level staff with example readings.

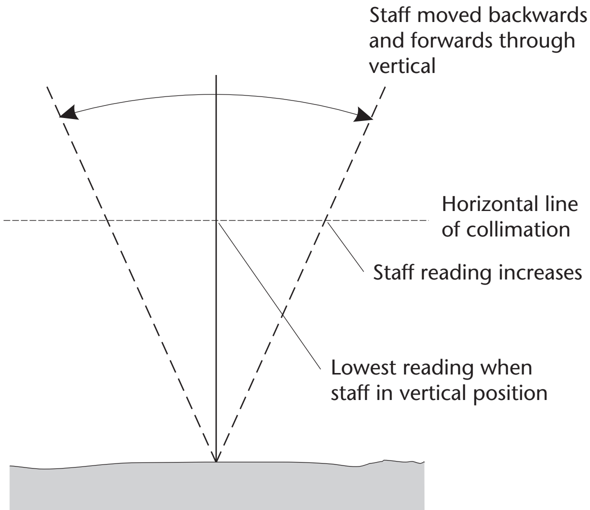

Figure 2.6 ● Taking vertical reading on staff.

mended in BS 4484 is used in the UK. This is shown in Figure 2.5 and can be read directly to 0.01 m and by estimation to 0.001 m.

Since the staff must be held vertically at each point where a height is to be measured most staffs are fitted with a circular bubble to help do this. If no bubble is available, the staff should be *slowly* moved back and forth through the vertical and the lowest reading noted – this will be the reading when the staff is vertical, as shown in Figure 2.6.

Automatic levels

The general features of the automatic level are shown in Figure 2.7. These instruments establish a horizontal plane at each point where they are set up and consist of a telescope and compensator. The *telescope* provides a magnified line of sight for taking measurements and the *compensator*, built into the telescope, ensures that the line of sight viewed through the telescope is horizontal even if the optical axis of the telescope is not exactly horizontal.

Surveying telescopes

The type of telescope used in automatic and other levels is very similar to that used in other surveying instruments such as theodolites and total stations and is shown in Figure 2.8. The following section refers specifically to the telescope of a level but is also applicable to measurements taken using other surveying instruments.

When looking through the *eyepiece* of the telescope, if it is correctly focused, a set of lines can be seen in the field of view and these provide a reference against which measurements are taken. This part of the telescope is called the *diaphragm* (or *reticule*)

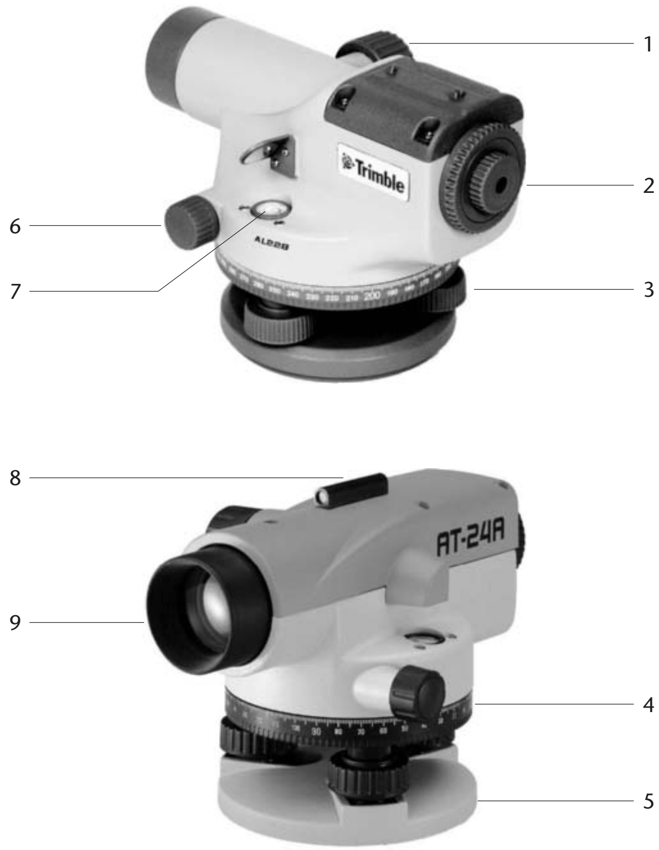

Figure 2.7 ● Automatic level: 1 focusing screw; 2 eyepiece; 3 footscrew; 4 horizontal circle; 5 base plate; 6 tangent screw (slow motion screw); 7 circular bubble; 8 collimator (sight); 9 object lens (courtesy Trimble & KOREC Group and Topcon).

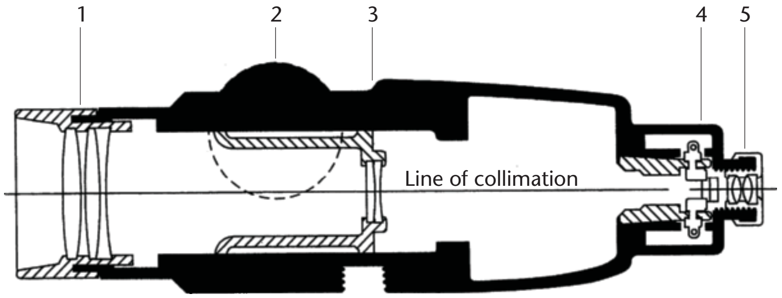

Figure 2.8 ● Surveying telescope: 1 object lens; 2 focusing screw; 3 focusing lens; 4 diaphragm; 5 eyepiece (courtesy Sokkia Topcon Co Ltd).

and this consists of a circle of plane glass upon which lines are etched, as shown in Figure 2.9. Conventionally, the pattern of vertical and horizontal lines is called the *cross hairs*.

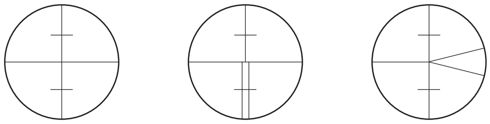

Figure 2.9 ● Diaphragm patterns.

The *object lens*, *focusing lens*, diaphragm and eyepiece are all mounted on the same optical axis and the imaginary line passing through the centre of the cross hairs and the optical centre of the object lens is called the *line of sight* or the *line of collimation* of the telescope. The diaphragm is held in the telescope by means of four adjusting screws so that the position of the line of sight can be moved within the telescope (see *Adjustment of the level* later in this section).

The action of the telescope is as follows. Light rays from the levelling staff (or target) pass through the object lens and are brought to a focus in the plane of the diaphragm by rotating the focusing screw. Rotating the focusing screw moves the focusing lens along the axis of the telescope. When the eyepiece is rotated, this also moves axially along the telescope and since it has a fixed focal point that lies outside the lens combination, its focal point can also be made to coincide with the plane of the diaphragm. Since the image of the levelling staff has already been focused on the diaphragm, an observer will see in the field of view of the telescope the levelling staff focused against the cross hairs. The image of the staff will also be highly magnified (see Figure 2.10) making accurate measurement of vertical distances possible over long distances.

A problem often encountered with outdoor optical instruments is water and dust penetration. In order to provide protection from these, the telescope and compensator compartment of some levels are sealed and filled, under pressure, with dry nitrogen gas. This is known as *nitrogen purging*, and since the gas is pressurised, water and dust are prevented from entering the telescope. The use of dry nitrogen also prevents lens clouding and moisture condensation inside the telescope.

Typical specifications for a surveying telescope for use in construction work are a magnification of up to about 30, a field of view of between 1 and 2° and a minimum focusing distance of 0.5–1.0 m. Some telescopes use *autofocusing*, where focusing is achieved by pressing a button in a similar manner to a camera (see also Section 5.3 under *Useful accessories* and Figure 5.23).

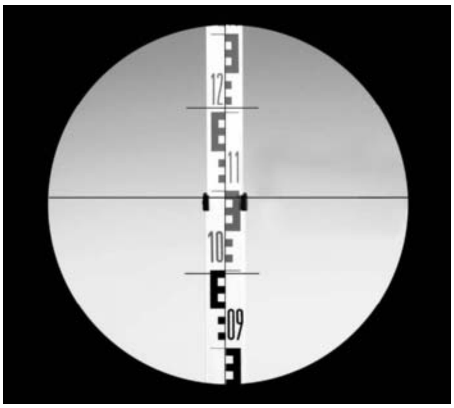

Figure 2.10 ● View of levelling staff through telescope (courtesy Leica Geosystems).

Parallax

For a surveying telescope to work correctly, the focusing screw has to be adjusted so that the image of the staff falls exactly in the plane of the diaphragm and the eyepiece must be adjusted so that its focal point is also in the plane of the diaphragm. Failure to achieve either of these settings results in a condition called *parallax*, and this can be a source of error when using levels, theodolites and total stations. Parallax can be detected by moving the eye to different parts of the eyepiece when viewing a levelling staff – if different parts of the staff appear against the cross hairs the telescope has not been properly focused and parallax is present, as shown in Figure 2.11.

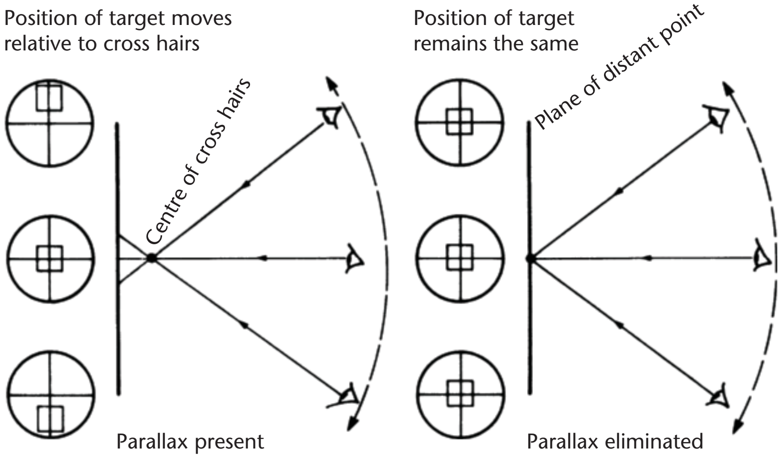

Figure 2.11 ● Parallax.

It is difficult to take accurate staff readings under these conditions, since the position of the cross hairs alters for different positions of the eye. For this reason, *it is essential that parallax is removed* before any readings are taken when using a level or any optical instrument with an adjustable eyepiece.

To remove parallax, a piece of white paper or a page from a field book is held in front of the object lens and the eyepiece is adjusted so that the cross hairs are in focus. The paper or field book is removed from in front of the object lens, and the staff at which readings are required is now sighted and brought into focus using the focusing screw. Next, the staff is observed whilst moving the eye up and down, and if it does not appear to move relative to the cross hairs then parallax has been eliminated (see Figure 2.11). If there is apparent movement then the procedure should be repeated. Once adjusted, it is not usually necessary to adjust the eyepiece again until a new set of readings is taken, say, on another day. For all levelling, the focusing screw has to be adjusted for each staff reading, as focus depends on the sighting distance.

The compensator

In an automatic level, the function of the compensator is to deviate a horizontal ray of light at the optical centre of the object lens through the centre of the cross hairs. This ensures that the line of sight (or collimation) viewed through the telescope is horizontal even if the telescope is tilted.

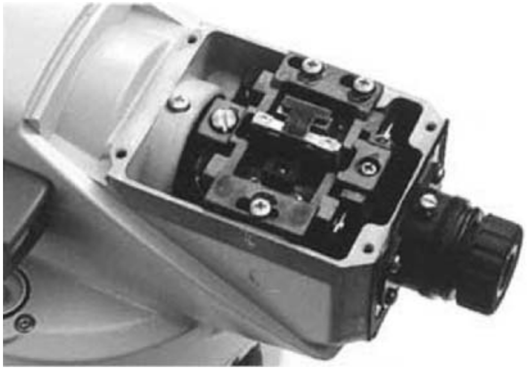

Figure 2.12 ● Compensator (courtesy Sokkia Topcon Co Ltd).

Whatever type of automatic level is used it must be levelled to within about 15' of the vertical through the level to allow the compensator to work. This is achieved by using the three footscrews together with the circular bubble.

Figure 2.12 shows a compensator and the position in which it is usually mounted in the telescope. The action of the compensator is shown in Figure 2.13, which has been exaggerated for clarity. The main component of the compensator is a prism which is assumed to be freely suspended within the telescope tube when the instrument has been levelled and which takes up a position under the influence of gravity according to the angle of tilt of the telescope. Provided the tilt is within the working range of the compensator, the prism moves to a position to counteract this and a horizontal line of sight (collimation) is always observed at the centre of the cross hairs.

The wires used to suspend a compensator are made of a special alloy to ensure stability and flexibility under rapidly changing atmospheric conditions, vibration and shock. The compensator is also screened against magnetic fields and uses some form of damping, otherwise it might be damaged when the level is in transit and might be affected by wind and vibration preventing readings from being taken.

Use of the automatic level

The first part of the levelling process is to set the tripod in position for the first reading, ensuring that the top of the tripod is levelled by eye after the tripod legs have been pushed firmly into the ground. Tripods normally have adjustable legs which enable their heads to be set level by eye if one leg goes further into the ground than the others. Following this, the level is attached to the tripod using the clamp provided and the circular bubble is centralised using the three footscrews. This ensures that the instrument is almost level and activates the compensator, which automatically moves to a position to establish a horizontal line of sight at the centre of the cross hairs. Therefore, at each set up, no further adjustment of the footscrews is required after the circular bubble has been set.

As with all types of level, parallax must be removed before any readings are taken.

In addition to the levelling procedure and parallax removal, a test should be made to see if the compensator is working before readings commence. One of the

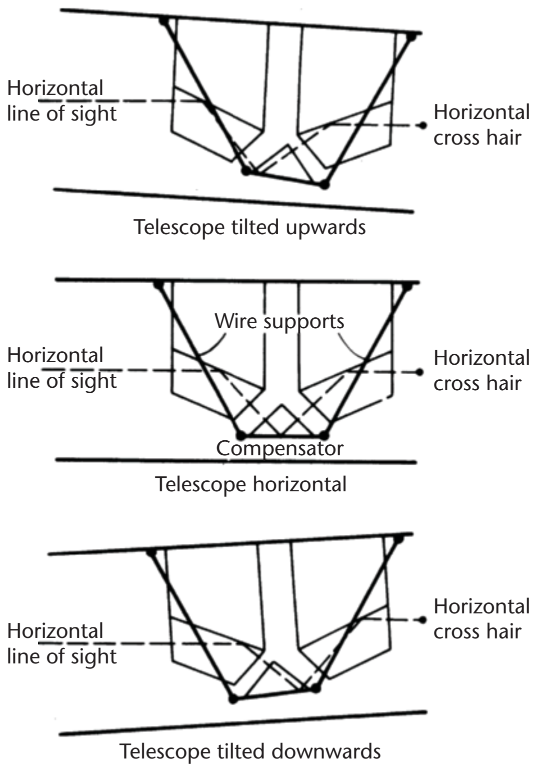

Figure 2.13 ● Action of compensator.

footscrews should be moved slightly off level and, if the reading to the levelling staff remains constant, the compensator is working. If the reading changes, it may be necessary to gently tap the telescope tube to free the compensator. When using some automatic levels, this procedure does not have to be followed as a small lever attached to the level enables the compensator to be checked. When a staff has been sighted, this lever is pressed (see Figure 2.14), and if the compensator is working the horizontal cross hair is seen to move and then return immediately to the horizontal line of sight. Additionally, some levels incorporate a warning device that gives a visual indication to an observer, in the field of view of the telescope, when the instrument is not level.

A problem sometimes encountered with levels that use a compensator is that machinery operating nearby will cause the compensator to vibrate, which in turn causes the image of the staff to appear to vibrate so that readings become very difficult to take. This problem can occur on construction sites, particularly where the site is narrow or constricted.

Tilting levels

Because of the popularity of the automatic and digital levels (which are discussed in the next section), the tilting level is rarely used on site these days. However, brief details are included here so that it can be compared with the others.

Figure 2.14 ● Compensator check (courtesy Leica Geosystems).

On this instrument, the telescope is not fixed to the base of the level and can be tilted a small amount in the vertical plane about a pivot placed below the telescope. The amount of tilt is controlled by a *tilting screw* which is usually directly underneath or next to the telescope eyepiece. Instead of a compensator, a tilting level will have a *spirit level* tube fixed to its telescope to enable a horizontal line of sight to be set. The spirit level tube (Figure 2.15) is a short barrel-shaped glass tube, sealed at both ends, that is partially filled with purified synthetic alcohol. The remaining space in the tube is an air bubble and there are a series of graduations marked on the glass top of the tube that are used to locate the relative position of the air bubble within the spirit level. The imaginary tangent to the surface of the glass tube at the centre of these graduations is known as the axis of the spirit level. When the bubble is centred in its run and takes up a position with its ends an equal number of graduations (or divisions) either side of the centre, the axis of the spirit level will be horizontal, as shown in Figure 2.15. By fitting a spirit level to a telescope such that its axis is parallel to the line of collimation, a horizontal line of sight can be set. This is achieved by adjusting the inclination of the telescope with the tilting screw until the bubble of the spirit level lies in the middle of its graduations.

In use, a tilting level is set up by attaching it to the tripod head and using the footscrews to centralise a circular bubble. As with the automatic level, this ensures that the instrument is almost level. Next, the telescope is turned until it is pointing in the direction in which the first staff reading is required and the tilting screw is rotated

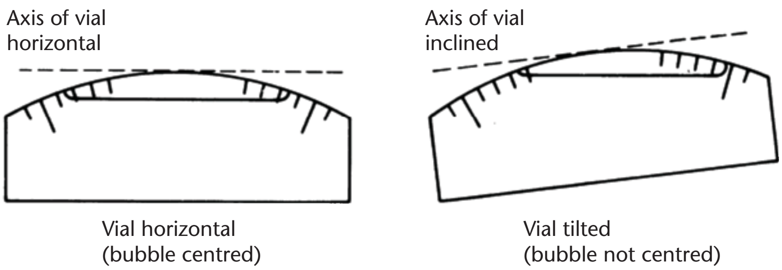

Figure 2.15 ● Spirit level tube (or vial).

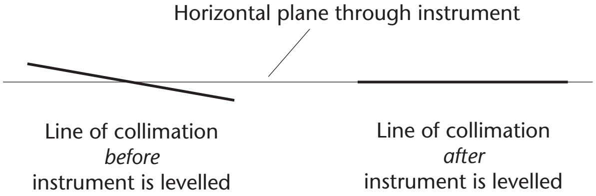

Figure 2.16 ● Principle of the tilting level.

until the spirit level bubble is brought to the centre of its run. This ensures that the line of collimation is horizontal, as shown in Figure 2.16, but only in the direction in which the reading is being taken. When the telescope is rotated to other directions, the bubble will change its position for each direction of the telescope because the instrument is not exactly levelled. Consequently, the tilting screw must be reset before every reading is taken and forgetting to do so is a common cause of errors when using a tilting level.

Digital levels

Shown in Figure 2.17, the digital level is not too dissimilar in appearance to an automatic or tilting level and the same features can be identified including the telescope with focus and eyepiece, footscrews with a base plate and so on. In use, it is set up in

Figure 2.17 ● Digital levels (courtesy Sokkia Topcon Co Ltd and Leica Geosystems).

Figure 2.18 ● Bar-coded levelling staff (courtesy Leica Geosystems).

the same way as an automatic level by attaching it to a tripod and centralising a circular bubble using the footscrews. A horizontal line of sight is then established by a compensator and readings could be taken in the same way as with an automatic level to a levelling staff, where all readings are taken and recorded manually.

However, this instrument has been designed to carry out all reading and data processing automatically via an on-board computer which is accessed through a display and keyboard. When levelling, a special bar-coded staff is sighted (see Figure 2.18), the focus is adjusted and a measuring key is pressed. There is no need to read the staff as the display will show the staff reading about two or three seconds after the measuring key has been pressed. When the bar-coded staff is sighted, it is interrogated by the level over a span of between 30 and 3000 mm using electronic image-processing techniques to produce a bar-coded image of the staff corresponding to the field of view of the telescope. The captured image is then compared by the on-board computer to the bar codes stored in the memory for the staff and when a match is found, this is the displayed staff reading. In addition to staff readings, it is also possible to display the horizontal distance to the staff with a precision of about 20–25 mm. All readings can be coded using the keyboard and as levelling proceeds, each staff reading and subsequently all calculations are stored in the level's internal memory.

In good conditions, a digital level has a range of about 100 m, but this can deteriorate if the staff is not brightly and evenly illuminated along its scanned section. The power supply for the digital level is standard AA or rechargeable batteries, which are capable of providing enough power for a complete day's levelling. If it is not possible to take electronic staff readings (because of poor lighting, obstructions such as foliage preventing a bar code from being imaged or loss of battery power), the reverse side of the bar-coded staff has a normal E type face and optical readings can be taken and entered manually into the instrument instead.

The digital level has many advantages over conventional levels since observations are taken quickly over longer distances without the need to read a staff or record anything

Figure 2.19 ● Removable memory card for digital level (courtesy Leica Geosystems).

by hand. This eliminates two of the worst sources of error from levelling – reading the staff incorrectly and writing the wrong value for a reading in the field book. As a digital level also calculates all the heights required, another source of error is removed from the levelling process – the possibility of making mistakes in calculations.

The data stored in a digital level can also be transferred to a removable memory card (see Figure 2.19) and then to a computer, where it can be processed further and permanently filed if required. To do this, the software provided by each manufacturer with the level can be used. As well as this, data can be transferred from the memory card directly into one of the many survey and design software packages now available. For on-site applications, design information can also be uploaded from a computer and memory card to the level for setting out purposes. All of the various options available for electronic data collection and processing are described in Section 5.4 for total stations.

Adjustment of the level

Whenever a level is set up, it is essential that the line of collimation, as viewed through the eyepiece, is horizontal. So far, the assumption has been made that once the circular bubble is centralised with the footscrews, the line of collimation is set exactly horizontal by the compensator and diaphragm (automatic and digital levels) or by centralising the bubble in the spirit level tube (tilting levels). However, because they are in constant use on site and therefore subjected to inevitable knocks and bumps, most levels are not in perfect adjustment and if horizontal readings are not being taken when it has been set up properly, a *collimation error* is present in the level. Since most levels will have a collimation error, some method is required to check this to determine if the error is within accepted limits. The most commonly used method is the *two-peg test* which should be carried out when using a new or different level for the first time and at regular intervals after this depending on how much the level has been used. Sometimes, the contract for a construction project will specify when the two-peg test should be carried out (say weekly) and will also specify a tolerance for the collimation error – because of this it is necessary to check all levels regularly when on site.

Referring to Figure 2.20, a two-peg test is carried out as follows.

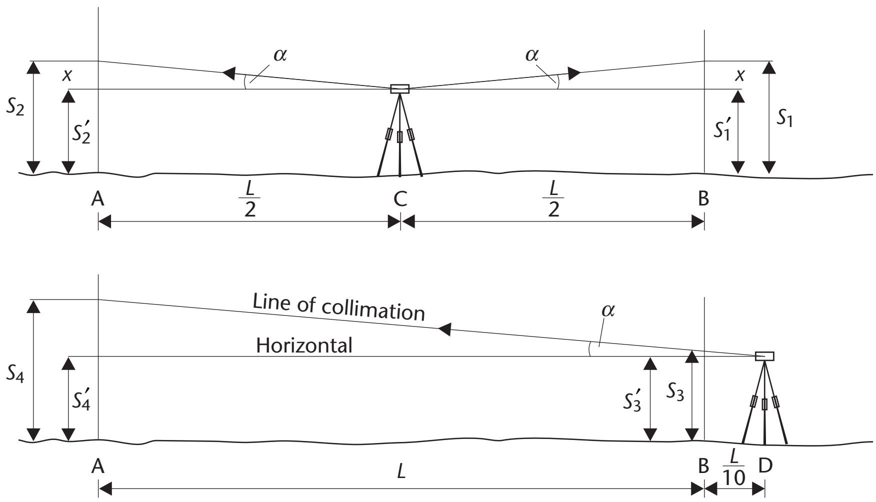

Figure 2.20 ● Two-peg test.

- On fairly level ground, two points A and B are marked a distance of L m apart. In soft ground, two pegs are used (hence the name of the test) and on hard surfaces, nails or paint can be used.
- The level is set up midway between the points at C and levelled. A levelling staff is placed at A and B in turn and staff readings S_1 (at B) and S_2 (at A) are taken.
- The two readings are

  $S_1 = (S_1' + x)$ and $S_2 = (S_2' + x)$

  where

  S_1' and S_2' are the staff readings that would have been obtained if the line of collimation was horizontal

  x is the error in each reading due to the collimation error, the effect of which is to tilt the line of sight by angle α.
- Although Figure 2.20 shows α and the line of collimation to be above the horizontal, it can be above or below. Since AC = CB, the error x in the readings S_1 and S_2 will be the same and the difference between readings S_1 and S_2 gives $S_1 - S_2 = (S_1' + x) - (S_2' + x) = S_1' - S_2'$, which gives the *true difference in height* between A and B. This demonstrates that if a collimation error is present in a level, the effect of this cancels out when height differences are computed provided readings are taken over equal sighting distances.
- The level is moved so that it is $\frac{L}{10}$ m from point B at D and readings S_3 and S_4 are taken (see Figure 2.20). The difference between readings S_3 and S_4 gives $S_3 - S_4$ = the *apparent difference in height* between A and B.
- If the level is in perfect adjustment $(S_1 - S_2) = (S_3 - S_4)$. However, it is usual that there is a difference between the true and apparent heights and since this has been measured over a distance of L m the collimation error for the level is given by

$$\text{collimation error } e = (S_1 - S_2) - (S_3 - S_4) \text{ per } L \text{ m} \quad (2.1)$$

- If the collimation error is found to be less than about ± 1 mm per 20 m (or some specified value) the level is assumed to be in adjustment.
- If the collimation error is found to be greater than about ± 1 mm per 20 m (or some specified value), the level has to be adjusted. To do this with the level still at point D, the horizontal reading S_4' that should be obtained at A is computed. The adjustment is then carried out, and this can be done by a number of different methods depending on the type of level being used. For automatic and digital levels, the diaphragm adjusting screws are loosened and the reticule is moved until reading S_4' is obtained. For some levels, the compensator has to be adjusted. A tilting level is adjusted by first turning the tilting screw to obtain S_4'. This causes the spirit level bubble to move from the centre of its run, so it is brought back to the centre by adjusting the vial. All of the mechanical adjustments described here for adjusting a level for collimation error are very difficult to do, especially on site, and if a level has an unacceptable collimation error, *it should be adjusted by a trained technician preferably under laboratory conditions*. This usually means returning it to the manufacturer.

Worked example 2.1: Two-peg test

Question

The readings obtained from a two-peg test carried out on an automatic level with a staff placed on two pegs A and B 50 m apart are:

- With the level midway between A and B
  Staff reading at A = 1.283 m　　Staff reading at B = 0.860 m
- With the level positioned 5 m from peg B on line AB extended
  Staff reading at A = 1.612 m　　Staff reading at B = 1.219 m

Calculate the collimation error of the level per 50 m sighting distance and the horizontal reading that should be observed on the staff at A with the level in position 5 m from B.

Solution

When solving problems of this type it is important that the numbering sequence shown in Figure 2.20 for staff readings is used otherwise an incorrect collimation error will be computed. In this case, the staff readings are identified as

$$S_1 = 0.860 \text{ m} \qquad S_2 = 1.283 \text{ m} \qquad S_3 = 1.219 \text{ m} \qquad S_4 = 1.612 \text{ m}$$

and from Equation (2.1):

$$\begin{aligned}\text{collimation error } e &= (0.860 - 1.283) - (1.219 - 1.612) \quad \text{per 50 m}\\ &= (-0.423 - (-0.393)) = \mathbf{-0.030 \text{ m per 50 m}}\end{aligned}$$

For the instrument in position 5 m from peg B, the horizontal reading that should have been obtained with the staff at A is

$$S'_4 = S_4 - [\text{collimation error per metre} \times \text{sighting distance DA}]$$
$$= 1.612 - \left[-\frac{0.030}{50}\right]55 = \mathbf{1.645\ m}$$

This is checked by computing S'_3, where

$$S'_3 = S_3 - [\text{collimation error per metre} \times \text{sighting distance DB}]$$
$$= 1.219 - \left[-\frac{0.030}{50}\right]5 = 1.222\ \text{m}$$

and

$$S'_3 - S'_4 = 1.222 - 1.645 = -0.423 = S_1 - S_2\ (\text{checks})$$

Reflective summary

With reference to levelling equipment, remember:

- Three different types of level are available – automatic, digital and tilting levels.
- The most widely used level on site is the automatic level but the digital level is gaining in popularity even though it is more expensive.
- All levels incorporate similar telescopes and to avoid errors occurring in staff readings, it is important that parallax is removed from a telescope before any readings are taken.
- Whatever type of level is used it must be checked for collimation error regularly.

2.3 Field procedure for levelling

After studying this section you should understand how height differences can be determined from staff readings and you should be familiar with the terms back sight, fore sight and intermediate sight, together with rise and fall in a levelling context. In addition, you should appreciate why it is important to start and finish all levelling at a bench mark.

How levelling is carried out

When a level has been correctly set up, the line or plane of collimation generated by the instrument coincides with or is very close to a horizontal plane. If the height of this plane is known, the heights of ground points can be found from it by reading a vertically held levelling staff.

In Figure 2.21, a level has been set up at point I_1 and readings R_1 and R_2 have been taken with the staff placed vertically in turn at ground points A and B. If the reduced level of A (RL_A) is known then, by adding staff reading R_1 to RL_A, the reduced level of

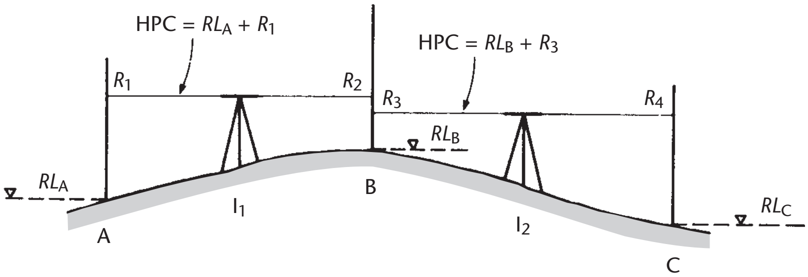

Figure 2.21 ● Principles of levelling.

the line of collimation at instrument position I_1 is obtained. This is known as the *height of the plane of collimation* (*HPC*) or the *collimation level*. This is given by

$$\text{collimation level at } I_1 = RL_A + R_1$$

In order to obtain the reduced level of point B (RL_B), staff reading R_2 must be subtracted from the collimation level to give

$$RL_B = \text{collimation level at } I_1 - R_2 = (RL_A + R_1) - R_2 = RL_A + (R_1 - R_2)$$

The direction of levelling in this case is from A to B and R_1 is taken with the level facing in the opposite direction to this. For this reason it is known as a *back sight* (BS). Since reading R_2 is taken with the level facing in the direction from A to B, it is called a *fore sight* (FS). The height change between A and B, both in magnitude and sign, is given by the difference between the staff readings taken at A and B. Since R_1 is greater than R_2 in this case, $(R_1 - R_2)$ is positive and the base of the staff has risen in moving from A to B. Because $(R_1 - R_2)$ is positive it is known as a *rise*.

The level is now moved to a new position I_2 so that the reduced level of C can be found. Reading R_3 is first taken with the staff still at point B but with its face turned towards I_2. This will be the back sight at position I_2 and the fore sight R_4 is taken with the staff at C. At point B, both a FS and a BS have been recorded consecutively, each from a different instrument position and this is called a *change point* (CP).

From the staff readings taken at I_2, the reduced level of C (RL_C) is calculated from

$$RL_C = RL_B + (R_3 - R_4)$$

The height difference between B and C is given both in magnitude and sign by $(R_3 - R_4)$. In this case, $(R_3 - R_4)$ is negative since the base of the staff has fallen from B to C. This time, the difference between the staff readings is known as a *fall*.

From the above, it can be seen that when calculating a rise or fall, this is always given by (preceding reading – subsequent reading) at a particular instrument position. If this is positive, a rise is obtained and if negative, a fall is obtained.

In practice, a BS is the first reading taken after the instrument has been set up and is always to a bench mark. Conversely, a FS is the last reading taken at an instrument position. Any readings taken between the BS and FS from the same instrument position are known as *intermediate sights* (IS).

A more complicated levelling sequence is shown in cross-section and plan in Figure 2.22, in which an engineer has levelled between two TBMs to find the reduced

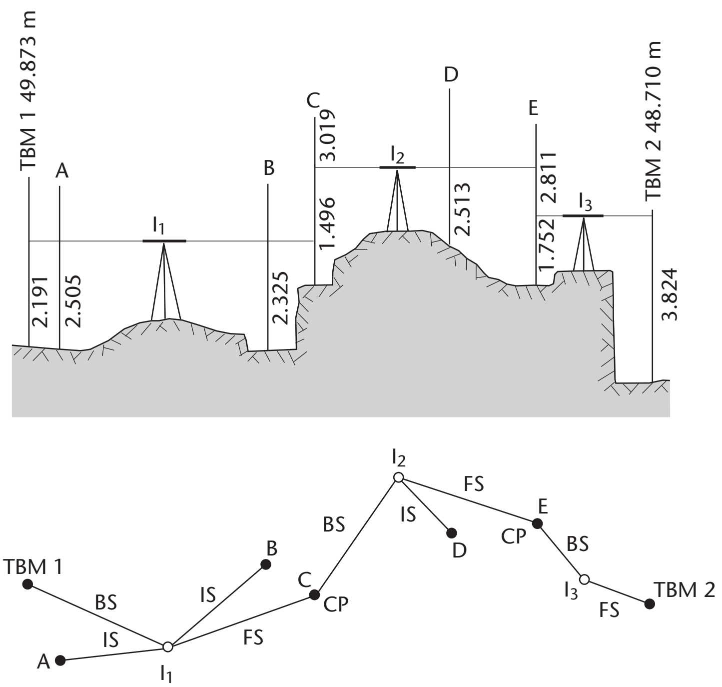

Figure 2.22 ● Line of levels.

levels of points A to E. The readings could have been taken with any type of level and the field procedure carried out to determine the reduced levels is as follows.

- The level is set up at some convenient position I_1 and a BS of 2.191 m is taken to TBM 1 with the foot of the staff being held on the TBM and the staff held vertically.
- The staff is then moved to points A and B in turn and readings are taken. These are intermediate sights of 2.505 m and 2.325 m respectively.
- A change point must be used in order to reach D owing to the nature of the ground. Therefore, a change point is chosen at C and the staff is moved to C and a FS of 1.496 m taken.
- While the staff remains at C, the instrument is moved to another position, I_2. A BS of 3.019 m is taken from the new level position to the staff at change point C.
- The staff is moved to D and E in turn and readings of 2.513 m (IS) and 2.811 m (FS) are taken where E is another CP.
- Finally, the level is moved to I_3, a BS of 1.752 m taken to E and a FS of 3.824 m taken to TBM 2.
- The final staff position is at a TBM. This is most important as all levelling fieldwork must start and finish at a bench mark, otherwise it is not possible to detect errors in the levelling.

> **Reflective summary**
>
> With reference to field procedures for levelling, remember:
>
> – Each reading must be identified as a back sight (BS), fore sight (FS) or intermediate sight (IS).
>
> – It is essential that all levelling starts and finishes at a bench mark.

2.4 Calculating reduced levels

After studying this section you should know how to calculate heights (reduced levels) by the rise and fall method and the height of plane of collimation (HPC) method. You should be aware that it is very important to carry out arithmetic checks on these calculations.

This section includes the following topics:

- The rise and fall method
- The height of collimation method

The rise and fall method

In this section, the heights of points A to E described in the previous section are computed by the *rise and fall* method.

Table 2.1 shows all the readings for the levelling sequence shown in Figure 2.22 recorded in a rise and fall field book. Each row or line of the field book corresponds to a staff position and this is confirmed by the entries made in the *Remarks* column. The calculation of reduced levels proceeds in the following manner, in which the reduced levels of points A to E are computed point-by-point starting at TBM 1.

- From the TBM 1 to A there is a small fall. A BS of 2.191 m has been recorded at TBM 1 and an IS of 2.505 m at A. So, for the fall from TBM 1 to A, the height difference is given by (2.192 – 2.505) = –0.314 m. The negative sign indicates a fall and this is entered in column for this on the line for point A. The fall is then subtracted from the RL of TBM 1 to obtain the initial reduced level of A as 49.873 – 0.314 = 49.559 m.
- The procedure is repeated and the height difference from A to B is given by (2.505 – 2.325) = +0.180 m. The positive sign indicates a rise and this is entered on the line for B. The RL of B is (RL_A + 0.180) = 49.739 m.
- The rise from B to C up to the first CP is (2.235 – 1.496) = +0.829 m from which the RL of C is (49.739 + 0.829) = 50.568 m.
- All the readings taken from I_1 have now been used and the calculation continues with the readings taken from I_2.

Table 2.1 ● Rise and fall method.

BS	IS	FS	Rise	Fall	Initial RL	Adj	Adj RL	Remarks
2.191					49.873		49.873	TBM 1 49.873
	2.505			0.314	49.559	+0.002	49.561	A
	2.325		0.180		49.739	+0.002	49.741	B
3.019		1.496	0.829		50.568	+0.002	50.570	C (CP)
	2.513		0.506		51.074	+0.004	51.078	D
1.752		2.811		0.298	50.776	+0.004	50.780	E (CP)
		3.824		2.072	48.704	+0.006	48.710	TBM 2 48.710
Checks								
6.962		8.131	1.515	2.684	48.704			
8.131			2.684		49.873			
–1.169			–1.169		–1.169			

Note: The date, observer and booker (if not the observer), the survey title, level number, weather conditions and anything else relevant should be recorded as well as the staff readings.

- The height change from C to D is obtained from (3.019 – 2.513) = + 0.506 m, which is a rise. Hence the RL of D is calculated from (50.568 + 0.506) = 51.074 m.
- The calculation continues as described above until the initial reduced level of TBM 2 is calculated.
- When the *Initial* RL column of the table has been completed, a check on the arithmetic involved is possible and must always be applied. This check is

  Σ BS – Σ FS = Σ RISES – Σ FALLS = LAST *Initial* RL – FIRST RL

  It is normal to enter these summations at the foot of each relevant column in the levelling table (see Table 2.1). Obviously, agreement must be obtained for *all three parts of the check* and it is stressed that this only provides a check on all the *Initial* RL calculations and does not provide an indication of the accuracy of the readings.

The difference between the calculated and known values of TBM 2 is –0.006 m. This is known as the *misclosure* of the levelling and it is the magnitude of this that gives an indication of the accuracy of the levelling.

If the misclosure is greater than the *allowable misclosure* then the levelling must be repeated, but if the misclosure is less than the allowable value then it is distributed throughout the reduced levels. In this case, the allowable misclosure is ± 9 mm and the levelling is acceptable (see Section 2.5 for a full explanation of this).

The usual method of correction is to apply an equal, but cumulative, amount of the misclosure to each instrument position, the sign of the adjustment being opposite to that of the misclosure. Since there is a misclosure of –0.006 m in this example a total adjustment of +0.006 m must be distributed. As there are three instrument positions, +0.002 m is added to the reduced levels found from each instrument position. In other words, it is assumed that an equal amount of the misclosure was caused every time the level was set up. So, in this case, all the RLs calculated from I_1 (apart from that of the initial bench mark) are adjusted by +0.002 m, all those calculated from I_2 are adjusted by +0.002 m for I_1 and +0.002 m for I_2, and all those calculated from I_3 are adjusted by +0.002 m for I_1, +0.002 m for I_2 and +0.002 m for I_3. The distribution is shown in the *Adj* (adjustment) column, in which the following cumulative adjustments have been applied: Levels A, B and C + 0.002 m, levels D and E + (0.002 + 0.002) = +0.004 m and TBM 2 + (0.002 + 0.002 + 0.002) = +0.006 m. No adjustment is applied to TBM 1 since this level cannot be altered. The adjustments are applied to the *Initial* RL values to give the *Adj* (adjusted) RL values in Table 2.1.

The height of collimation method

Table 2.2 shows the field book for the reduction of the levelling of Figure 2.22 by the height of collimation method. This way of reducing levels is based on the HPC being calculated for each instrument position and proceeds as follows.

- If the BS reading taken to TBM 1 is added to the RL of this bench mark, then the HPC for the instrument position I_1 will be obtained. This will be 49.873 + 2.191 = 52.064 m and this is entered in the HPC column on the same line as the BS.
- To obtain the initial reduced levels of A, B and C the staff readings to those points are now subtracted from the HPC. The relevant calculations are

  RL of A = 52.064 – 2.505 = 49.559 m

  RL of B = 52.064 – 2.325 = 49.739 m

  RL of C = 52.064 – 1.496 = 50.568 m

Table 2.2 ● Height of collimation method.

BS	IS	FS	HPC	Initial RL	Adj	Adj RL	Remarks
2.191			52.064	49.873		49.873	TBM 1 49.873
	2.505			49.559	+0.002	49.561	A
	2.325			49.739	+0.002	49.741	B
3.019		1.496	53.587	50.568	+0.002	50.570	C (CP)
	2.513			51.074	+0.004	51.078	D
1.752		2.811	52.528	50.776	+0.004	50.780	E (CP)
		3.824		48.704	+0.006	48.710	TBM 2 48.710
Checks							
6.962	7.343	8.131		300.420			
7.343 + 8.131 + 300.420 = 315.894							
[52.064 × 3] + [53.587 × 2] + [52.528] = 315.894							
6.962		8.131		48.704			
8.131				49.873			
–1.169				–1.169			

Note: The date, observer and booker (if not the observer), the survey title, level number, weather conditions and anything else relevant should be recorded as well as the staff readings

- At point C, a change point, the instrument is moved to position I_2 and a new HPC is established. This collimation level is obtained by adding the BS at C to the RL found for C from I_1. For position I_2, the HPC is 50.568 + 3.019 = 53.587 m. The staff readings to D and E are now subtracted from this to obtain their reduced levels.
- The procedure continues until the *Initial* RL of TBM 2 is calculated and the misclosure found as before. With the *Initial* RL column in the table completed, the following check can be applied.

  Σ BS – Σ FS = LAST *Initial* RL – FIRST RL

  However, this only verifies the reduced levels calculated using BS and FS readings. To check the reduced levels calculated from IS readings, a second check is used and is given by

  Σ IS + Σ FS + Σ RLs except first = Σ [each HPC × number of applications]

  Table 2.2 gives

  Σ IS + Σ FS + Σ RLs except first = 7.343 + 8.131 + 300.420 = 315.894 m

  The first HPC of 52.064 m has been used *three* times to calculate the reduced levels of A, B and C. The second of 53.587 m has been used *twice* to calculate the reduced levels of D and E and the last HPC of 52.528 m has been used *once* to close the levels onto TBM 2. This gives the second part of this check as

  [52.064 × 3] + [53.587 × 2] + [52.528 × 1] = 315.894 m
- After applying the check, any acceptable misclosure is distributed as for the rise and fall method.

Reflective summary

With reference to calculating reduced levels, remember:

- There are two ways of calculating reduced levels – the rise and fall method and the height of plane of collimation (HPC) method.
- The arithmetic checks MUST be done for all levelling calculations.
- When establishing the heights of new TBMs and other important points, only BS and FS readings should be taken and the rise and fall method of calculation should be used.
- The HPC method of calculation can be much quicker when a lot of intermediate sights have been taken and it is a good method to use when mapping or setting out where many readings are often taken from a single instrument position.
- A disadvantage of the HPC method is that the check on reduced levels calculated from IS readings can be lengthy and there is a tendency for it to be omitted.

2.5 Precision of levelling

After studying this section you should be able to determine the allowable misclosure for different types of levelling and you should realise how this helps to decide whether a line of levels is accepted or is rejected.

How good is my levelling?

In the previous section, a misclosure of –6 mm was obtained for the levelling by comparing the reduced level of the closing bench mark TBM 2 (48.710 m) with its *Initial* RL obtained from staff readings (48.704 m). By comparing the two reduced levels for TBM 2 in this way, an assessment of the quality or precision of the levelling can be made and it is usual to check that the misclosure obtained is better than some specified value called the *allowable misclosure*.

On construction sites and other engineering projects, levelling is usually carried out over short distances and it can include a lot of instrument positions. For this type of work, the *allowable misclosure* for levelling is given by

$$\textit{allowable misclosure} = \pm\, m\sqrt{n}$$

where m is a constant and n is the number of instrument positions. A value often used for m is 5 mm.

When the misclosure obtained from staff readings is compared to the *allowable misclosure* and it is found that the misclosure is greater than the allowable value, the levelling is rejected and has to be repeated. If the misclosure is less than the allowable value, the misclosure is distributed between the instrument positions as described in the previous section. For the levelling given in Tables 2.1 and 2.2, the misclosure is –6 mm, the allowable misclosure is given by (with m = 5 mm) $\pm 5\sqrt{3} = \pm 9$ mm and the levelling has been accepted.

When assessing the precision of any levelling by this method, it may be possible for a site engineer to use a value of m based on site conditions. For example, if the reduced levels found are to be used to set out earthwork excavations, the value of m might be 30 mm but for setting out steel and concrete structures, the value of m might be 3 mm. Values of m may be specified as tolerances in contract documents or where they are not given, may simply be chosen by an engineer based on experience.

Specifications for levelling are also given in BS 5964: *Building setting out and measurement* and in the ICE Design and Practice Guide *The management of setting out in construction*. Typical accuracies expected for levelling are also listed in BS 5606: *Guide to accuracy in building* and it possible to evaluate the accuracy of anyone levelling by using BS 7334: *Measuring instruments for building construction*. For further details of these, see *Further reading and sources of information* at the end of the chapter. The precision of levelling is also discussed in Chapter 9.

Reflective summary

With reference to the precision of levelling, remember:

- An allowable misclosure should always be computed and a decision made to accept or reject the levelling.
- The allowable misclosure varies according to the number of times the level is set up and how the levels obtained are going to be used.
- If the levelling is accepted, the reduced levels obtained should be adjusted.
- If it is rejected, the levelling must be repeated – never try to correct or invent a reading to make a line of levels close.

2.6 Sources of error in levelling

After studying this section you should be aware of the various sources of error that can occur in levelling and you should appreciate how it is possible to minimise or even eliminate some of these by adopting sensible field procedures.

This section includes the following topics:

- Errors in the equipment
- Field or on-site errors
- The effects of curvature and refraction on levelling
- How to reduce the chance of errors occurring

Errors in the equipment

Collimation error

This can be a serious source of error in levelling if sight lengths from one instrument position are not equal, since the collimation error is proportional to the difference in these. So, in all types of levelling, sight lengths should be kept equal, particularly back sights and fore sights and before using any level it is advisable to carry out a two-peg test to ensure the collimation error is within acceptable limits.

Compensator not working

For an automatic or digital level, the compensator is checked by moving a footscrew slightly off level, by tapping the telescope gently or by pushing the compensator check lever (if fitted) to ensure that a reading remains constant. If any of these checks fail, the compensator is not working properly and the instrument must be returned to the manufacturer for repair.

Parallax

This effect must be eliminated before any staff readings are taken.

Defects of the staff

The base of the staff should be checked to see if it has become badly worn – if this is the case then the staff has a *zero error*. This does not affect height differences if the same staff is used for all the levelling, but introduces errors if two staffs are being used for the same series of levels.

When using a multi-section staff, it is important to ensure that it is properly extended by examining the graduations on either side of each section as it is extended. If any of the sections become loose, the staff should be returned for repair.

Tripod defects

The stability of tripods should be checked before any fieldwork commences by testing to see if the tripod head is secure, that the metal shoes at the base of each leg are not loose and that, once extended, the legs can be tightened sufficiently.

Field or on-site errors

Staff not vertical

Since the staff is used to measure a vertical difference between the ground and the plane of collimation, failure to hold the staff vertical will give incorrect readings. If the staff is fitted with a circular bubble to aid it being held vertically, this should be checked at frequent intervals and adjusted if necessary. However, a better procedure is to rock the staff slowly back and forth through the vertical wherever possible and to note the lowest reading, as shown in Figure 2.6.

Unstable ground

When the instrument is set up on soft ground and bituminous surfaces on hot days, an effect often overlooked is that the tripod legs may sink into the ground or rise slightly whilst readings are being taken. This alters the height of collimation and it is advisable to choose firm ground on which to set up the level and tripod, and to ensure that the tripod shoes are pushed well into the ground.

Similar effects can occur with the staff, and for this reason it is particularly important that change points should be at stable positions such as manhole covers, kerbstones, concrete surfaces, and so on. This ensures that the base of the staff remains at the same height in between a FS and BS.

For both the level and staff, the effect of soft or unstable ground is greatly reduced if readings are taken in quick succession.

Handling the instrument and tripod

As well as vertical displacement, the plane of collimation of a level may be altered for any set-up if the tripod is held or leant against. When levelling, avoid contact with the tripod and only use the level by light contact through the fingertips. If at any stage the tripod is disturbed, it will be necessary to relevel the instrument and to repeat all the readings taken from that instrument position.

Instrument not level

For automatic and digital levels this source of error is unusual, but for a tilting level in which the tilting screw has to be adjusted for each reading, this is a common mistake. The best procedure here is to ensure that the main bubble is centralised before and after a reading is taken.

Reading and booking errors

Many mistakes can be made during the booking of staff readings taken with an automatic or tilting level, and the general rule is that staff readings must be carefully entered into the levelling table or field book *immediately after reading*. As already noted, readings taken with a digital level are automatically stored by the instrument and there is no need for the operator to record anything by hand – this gives the digital level an advantage over automatic and tilting levels.

Weather conditions

In strong winds, a level can become unusable because the line of sight is always moving and it also very difficult to hold the staff steady. For these reasons, it is not possible to take reliable readings under these conditions which should be avoided when levelling.

In hot weather, the effects of refraction are serious and produce a shimmering effect near ground level that makes it very difficult to read the bottom metre of the staff.

The effects of curvature and refraction on levelling

In Section 2.1 it was shown that when a level is set up, it defines a horizontal line for measurement of height differences. In Figure 2.23 a level is shown set up at point A and it can be seen that the level and horizontal lines through the instrument diverge because level lines follow the curvature of the Earth which is defined as mean sea level. If not accounted for, this is a possible source of error in levelling since all readings taken at A are observed along the horizontal line instead of the level line.

However, for most levelling applications, sighting distances are relatively short and the curvature correction for a length of sight of 50 m is less than 1 mm. Consequently, when levelling, the difference between a horizontal line and a level line is small enough to be ignored.

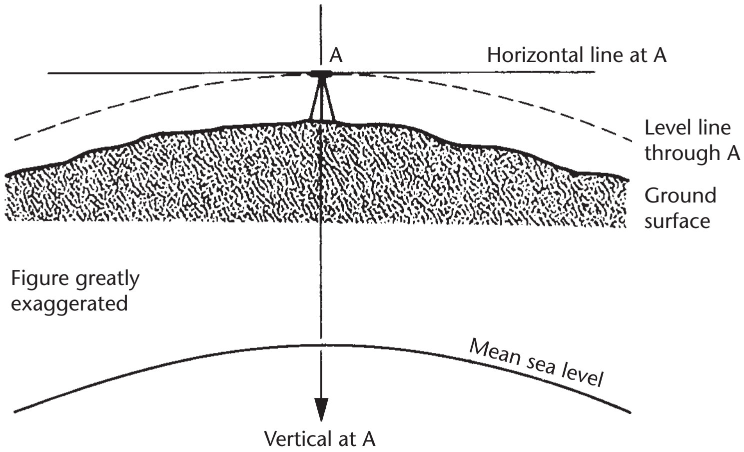

Figure 2.23 ● Curvature.

The effect of atmospheric refraction on a line of sight is to bend it either towards or away from the Earth's surface. This has a value that can vary considerably, especially close to the ground, but it can be ignored for most levelling, where the length of sight is seldom greater than 50 m.

Whatever sight lengths are used, the effects of curvature and refraction will cancel if the sight lengths are equal. However, the effects of curvature and refraction cannot be ignored when measuring heights over long distances using total stations – this is discussed in Section 5.6.

How to reduce the chance of errors occurring

When levelling, the following procedures should be used if many of the sources of error are to be avoided.

- Levelling should always start and finish at bench marks so that misclosures can be detected. When only one bench mark is available, levelling lines must be run in loops, starting and finishing at the same bench mark.
- Where possible, all sight lengths should be below 50 m to enable the staff to be read accurately.
- The staff must be held vertically by suitable use of a circular bubble or by rocking the staff and noting the minimum reading.
- BS and FS lengths should be kept equal for each instrument position to eliminate the effects of any collimation error. For engineering applications, many IS readings may be taken from each setup. Under these circumstances it is important that the level has no more than a small collimation error.
- For automatic and tilting levels, staff readings should be booked immediately after they are observed and important readings, particularly at change points, should be checked. Use a digital level where possible as it takes staff readings automatically.

- The rise and fall method of reduction is preferable when heighting reference or control points where few, if any, IS readings are taken and the HPC method is preferable when setting out where a lot of IS readings are often taken.

Reflective summary

With reference to the sources of error in levelling, remember:

– *Instrumental errors* can be avoided by proper maintenance and adjustment of equipment.

– *Field errors* are caused by carelessness when on site – make sure recommended procedures are followed to reduce the chance of these occurring.

– Mistakes in observing and recording staff readings are a matter of experience – the best way to avoid these is to *read – write it down – read* at each staff position.

– Because it records automatically, the digital level is capable of taking staff readings that are error-free.

– Levelling in extreme weather conditions can often be difficult and is not recommended.

2.7 Other levelling methods

After studying this section you should understand how it is possible to determine the heights of elevated points using inverted staff methods and you should be aware of the technique used to carry a line of levels over a wide gap avoiding any instrumental errors that may be present in the level.

This section include the following topics:

- Inverted staff
- Reciprocal levelling

Inverted staff

Occasionally, it may be necessary to determine the heights of points such as a ceiling or the soffit of a bridge, underpass or canopy. Usually, these points will be above the plane of collimation of the level. To obtain the reduced levels of these points, the staff is held upside down in an inverted position with its base on the elevated points. When booking an inverted staff reading, it is entered into the levelling table with a *minus sign*, the calculation proceeding in the normal way taking this sign into account.

An example of a levelling line including inverted staff readings is shown in Figure 2.24 and the relevant calculations for this are in Table 2.3.

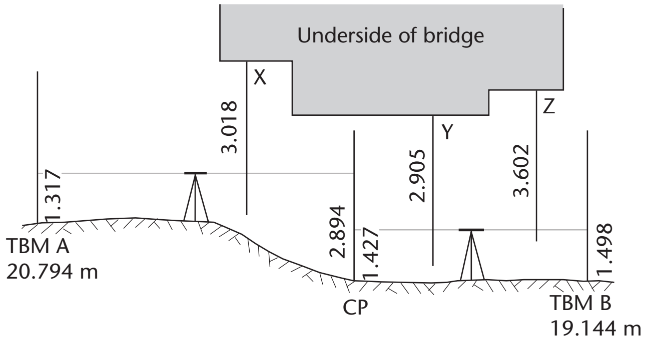

Figure 2.24 ● Inverted staff levelling.

Table 2.3 ● Inverted staff readings.

BS	IS	FS	Rise	Fall	Initial RL	Adj	Adj RL	Remarks
1.317					20.794		20.794	TBM A 20.794
	–3.018		4.335		25.129	–0.001	25.128	X
1.427		2.894		5.912	19.217	–0.001	19.216	CP
	–2.905		4.332		23.549	–0.002	23.547	Y
	–3.602		0.697		24.246	–0.002	24.244	Z
		1.498		5.100	19.146	–0.002	19.144	TBM B 19.144
Checks								
2.744		4.392	9.364	11.012	19.146			
4.392			11.012		20.794			
–1.648			–1.648		–1.648			

Note: The date, observer and booker (if not the observer), the survey title, level number, weather conditions and anything else relevant should be recorded as well as the staff readings

Each inverted staff reading is denoted by a minus sign and the rise or fall computed accordingly. For example, the rise from TBM A to point X is 1.317 – (–3.018) = 4.335 m. Similarly, the fall from point Z to TBM B is –3.602 – (1.498) = –5.100 m.

An inverted staff position *must not be used as a change point* because it is often difficult to keep the staff vertical and to keep its base in the same position for more than one reading.

Reciprocal levelling

For all levelling, true differences in height between pairs of points are obtained by ensuring that their sight lengths are equal. This eliminates the effect of any collimation error and also the effects of curvature and refraction.

However, there are certain applications in engineering and site work when it may not be possible to take staff readings with equal sight lengths, as, for instance, when a

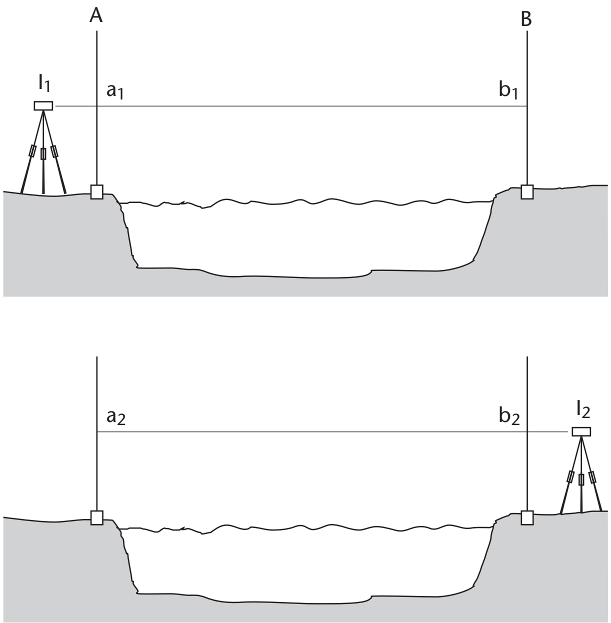

Figure 2.25 ● Reciprocal levelling.

line of levels has to be taken over a gap such as a river or ravine. In these cases, the technique of *reciprocal levelling* can be used.

Figure 2.25 shows two points A and B on opposite sides of a river. To obtain the true difference in height between A and B a level is placed at I_1 close to A on one side of the river and a levelling staff is held vertically at A and B in turn. Staff readings of a_1 at A and b_1 at B are taken and the level is then moved to the other side of the river to position I_2 where readings a_2 and b_2 are taken.

The difference in height between A and B is obtained by treating each reading at A as a BS and each reading at B as a FS. This gives the height difference ΔH_{AB} as $(a_1 - b_1)$ or $(a_2 - b_2)$, but with a BS and FS taken over very unequal sight lengths from each instrument position. Since staff readings a_1 and b_2 are taken over short distances, it is assumed that the effect of any collimation error in the level and the effects of curvature and refraction on these readings are small and can be ignored. This will not be the case for staff readings b_1 and a_2, which have been taken over long distances. However, since these readings have been taken over the same distances with the same level, the combined effect ε of the collimation error and curvature and refraction will be the same in both readings. The corrected staff readings are $(b_1 + \varepsilon)$ and $(a_2 + \varepsilon)$ and the true difference in height between A and B is given by the mean of the two observed differences from I_1 and I_2 as

$$\Delta H_{AB} = \tfrac{1}{2}[(a_1 - [b_1 + \varepsilon]) + ([a_2 + \varepsilon] - b_2)] = \tfrac{1}{2}[(a_1 - b_1) + (a_2 - b_2)]$$

in which the effects of the collimation error and curvature and refraction have cancelled.

For this type of work, several sets of readings are taken with the instrument being re-levelled in a slightly different position for each set. The average values for the staff readings are then used to compute ΔH_{AB}. When carrying out this procedure with one level, the two sets of observations must follow each other as soon as possible so that refraction effects are the same and are minimised. Where this is not possible, two levels have to be used simultaneously. As it is very unlikely that the two levels will have the same collimation error, the true height difference will not be obtained. This problem is overcome by interchanging the levels and repeating the whole procedure and then taking the mean value for the height difference from these. Another potential difficulty with this method of height measurement is its reliability as it becomes difficult to read a levelling staff as the sighting distance increases beyond about 100 m. It is recommended that, for distances of more than about 100 m, another method, such as trigonometrical heighting or GNSS, should be used to transfer height across a wide gap.

Reflective summary

With reference to other levelling methods, remember:

– The inverted staff method is very useful on site for finding the heights of points up to about 4 or 5 m above the level (the length of a levelling staff) but because it can be difficult to hold the staff in an inverted position the accuracy may not be as good as ordinary levelling.

– Reciprocal levelling can be used very effectively to transfer levels across an inaccessible gap but its accuracy is governed by how well a levelling staff can be read at long distances.

Exercises

2.1 Explain how parallax occurs and describe a procedure for removing it from a level.

2.2 Explain the difference between a level line and a horizontal line.

2.3 Why is it necessary to try and keep sight lengths as equal as possible when levelling?

2.4 What is the height datum in mainland Great Britain called and what is an OSBM?

2.5 What are the advantages of using a digital level compared to an automatic level?

2.6 Describe a test that can be carried out to determine the collimation error of a level.

2.7 Discuss the circumstances under which the rise and fall or HPC method would be used for reducing levels.

2.8 Some levelling is carried out on site where four instrument positions were used. What would be the allowable error for this levelling if it was to be used for setting out (a) a steel structure and (b) earthworks?

2.9 State the sources of error that can occur in levelling and describe how these can be minimised.

2.10 Explain why it is not advisable to have a change point at an inverted staff position.

2.11 At what distance does Earth curvature have a value of 10 mm? *Note: See Section 5.6 for curvature correction formula.*

2.12 The observations listed below were recorded when testing the adjustment of an automatic level. Use them to calculate the collimation error in the level.

Level position	Staff position	Sighting distance	Staff reading
I_1	A	25 m	1.225 m
I_1	B	25 m	1.090 m
I_2	A	55 m	1.314 m
I_2	B	5 m	1.155 m

2.13 A digital level was checked for collimation error using a two-peg test and the following results were obtained.

With the level midway between two pegs B1 and B2 which are 40 m apart:
Staff reading at B1 = 1.476 m Staff reading at B2 = 1.432 m

Level set up 10 m from B2 along the line B1–B2 extended:
Staff reading at B1 = 1.556 m Staff reading at B2 = 1.472 m

Calculate the collimation error in the level and the readings that would have been obtained with the level in the second position close to B2 had it been in perfect adjustment.

2.14 The readings shown below were taken to find the heights of pegs A–D. Calculate adjusted reduced levels for the pegs.

BS	IS	FS	Remarks
1.603			TBM 40.825 m
	1.001		Peg A
1.761		1.367	CP
	1.297		Peg B
1.272		1.203	CP
	0.910		Peg C
1.979		2.291	CP
0.772		0.646	Peg D
		3.030	TBM 39.685 m

2.15 For the levelling shown below, calculate adjusted reduced levels for all points in the level table.

BS	IS	FS	Remarks
1.832			TBM 62.117 m
2.150		2.379	Change point
	1.912		A
	1.949		B
	2.630		C
1.165		1.539	D
2.381		2.212	Change point
	2.070		E
	2.930		F
	0.954		G
	2.425		H
		0.879	TBM 62.629 m

2.16 Select a suitable method and reduce the levels given below.

BS	IS	FS	Remarks
1.729			TBM 71.025 m
	0.832		Peg at chainage 200 m
	0.971		210 m
	1.002		220 m
	1.459		230 m
	1.031		240 m
	1.600		250 m
	1.621		260 m
	2.138		270 m
	2.076		280 m
		1.730	TBM 71.025 m

2.17 The extract given below is for levels taken between two TBMs. Calculate adjusted reduced levels for all entries in the book.

BS	IS	FS	Remarks
1.592			TBM 31.317 m
1.675		2.052	CP
1.354		1.704	CP
1.326		0.907	Peg A
	1.379		Peg B
	1.384		Peg C
	1.406		Peg D
0.940		1.315	Peg E
0.832		1.507	CP
	–2.938		Soffit 1
	–2.833		Soffit 2
	–2.717		Soffit 3
		1.546	TBM 30.007 m

Note: The readings having minus signs were taken with an inverted staff.

2.18 Reciprocal levelling involving a single level was used to transfer height across a river between two points R1 and R2 70 m apart. Using the following readings, calculate the height change from R1 to R2 and the collimation error in the level.

Level set up close to R1: Staff reading on R1 = 1.582 m
Staff reading on R2 = 0.792 m

Level set up close to R2: Staff reading on R1 = 2.112 m
Staff reading on R2 = 1.336 m

Further reading and sources of information

For assessing the accuracy of levelling, consult

BS 5606: 1990 *Guide to accuracy in building* (British Standards Institution [BSI], London). BSI web site http://www.bsi-global.com/.

BS 5964-1: 1990 (ISO 4463-1: 1989) *Building setting out and measurement. Methods of measuring, planning and organisation and acceptance criteria* (British Standards Institution, London). BSI web site http://www.bsi-global.com/.

BS 7334-3: 1990 (ISO 8322-3:1989) *Measuring instruments for building construction. Methods for determining accuracy in use: optical levelling instruments*. (British Standards Institution, London). BSI web site http://www.bsi-global.com/.

ISO 17123–2: 2001 *Optics and optical instruments – Field procedures for testing geodetic and surveying instruments – Part 2: Levels* (International Organization for Standardization [ISO], Geneva). ISO web site http://www.iso.org/.

For general guidance and assessing the accuracy of levelling on site refer to

ICE Design and Practice Guide (1997) *The management of setting out in construction.* Thomas Telford, London.

For information on OSBMs and GNSS-based heights refer to:

RICS (2006) *Virtually level*. Available to download at http://www.rics.org/mappp/.

For the latest information on the equipment available, visit the following web sites:

http://www.leica-geosystems.com/
http://www.nikon-trimble.com/
http://www.pentaxsurveying.com/
http://www.sokkia.net/
http://www.topcon.eu/
http://www.trimble.com/

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Levelling Worksheets

1. (2.7) The following readings were taken with a level and a 4.25-m staff:
   0.683, 1.109, 1.838, 3.398 [3.877 and 0.451] CP, 1.405, 1.896, 2.676 BM (102.120 AOD), 3.478 [4.039 and 1.835] CP, 0.649, 1.707, 3.722
   Draw up a level book and reduce the levels by
   (a) R-and-F.
   (b) Height of collimation.
   What error would occur in the final level if the staff had been wrongly extended and a plain gap of 12 mm occurred at the 1.52-m section joint?

   [Answer: Parts (a) and (b) are self checking. Error in final level = zero.] (Hint: all readings greater than 1.52 m will be too small by 12 mm. Error in final level will be calculated from benchmark only.)

2. The following staff readings in metres were obtained when levelling along the centre-line of a straight road ABC.

Back Sight (BS)	Intermediate Sight (IS)	Fore sight (FS)	Remarks
2.405			A (RL = 250.05 m AOD)
1.954		1.128	CP
0.619		1.466	B
	2.408		D
	-1.515		E
1.460		2.941	CP
		2.368	C

D is the highest point on the road surface beneath a bridge crossing over the road at this point and the staff was held inverted on the underside of the bridge girder at E, immediately above D. Reduce the levels correctly by an approved method, applying the checks, and determine the headroom at D. If the road is to be re-graded so that AC is a uniform gradient, what will be the new headroom at D? The distance AD = 240 m and DC = 60 m.

[Answer: 3.923 m, 5.071 m]